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What are effective a_1 and a_2 in two-body hadronic decays of D and B mesons?

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Abstract

Through a specific example of two-body color-favored charm decay, $D_s^+ \to \phi\pi^+$, we illustrate how an effective and complex (unitarized) a_1, denoted by $a_1^{U,eff}$, may be defined such that it includes nonfactorized, annihilation and inelastic final state interaction (fsi) effects. The procedure can be generalized to color-suppressed processes to define an effective, and complex $a_2^{U,eff}$. We determine $|a_1^{U,eff}|$ and, where relevant, $|a_2^{U,eff}|$ for $D \to \bar{K}\pi, \bar{K}\rho, \bar{K}^*\pi, D_s^+ \to \eta\pi^+$, $\eta'\pi^+$, $\eta\rho^+, \eta'\rho^+$, and for $B^0 \to D^-\pi^+$ and $D^-\rho^+$ from the hadronic and semileptonic decay data.

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1 Introduction

We begin with some definitions relevant to the hadronic decays of charmed mesons. The effective Hamiltonian for Cabibbo-favored charmed decays is given by,

$$H_w^{eff}(\Delta C = \Delta S = -1) = \tilde{G}_F \left\{C_1(\bar{s}c)(\bar{u}d) + C_2(\bar{s}d)(\bar{u}c)\right\} \ , \tag{1}$$

where $\tilde{G}_F \equiv \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^*$ and the brackets $(\bar{s}c)$ etc. represent color-singlet (V-A) hadronic currents with appropriate flavors and V_{cs} etc are the Cabibbo-Kobayashi-Maskawa (CKM) mixing parameters. G_F is the Fermi Weak coupling constant. C_1 and C_2 are the Wilson coefficients [1] for which we take the values,

$$C_1 = 1.26 \pm 0.04, \qquad\qquad C_2 = -0.51 \pm 0.05 \ . \tag{2}$$

The central values are taken from [2]; the error assignments are ours. The parameters a_1 and a_2 are defined as follows,

$$a_{1,2} = C_{1,2} + \frac{1}{N_c} C_{2,1} \ , \tag{3}$$

where N_c is the number of colors.

The formula corresponding to (1) and (2) for Cabibbo-favored bottom decays are,

$$H_w^{eff}(\Delta B = \Delta C = -1) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[C_1(\bar{c}b)\left\{(\bar{d}u) + (\bar{s}c)\right\} + C_2\left\{(\bar{c}u)(\bar{d}b) + (\bar{c}c)(\bar{s}b)\right\}\right] \ , \tag{4}$$

and [2],

$$C_1 = 1.12 \pm 0.01, \qquad\qquad C_2 = -0.27 \pm 0.03 \ . \tag{5}$$

Again, the error assignments are ours.

In phenomenology as practiced until recently, it was found [3] that the choice $a_{1,2} = C_{1,2}$ worked as a reasonable approximation in the factorization scheme for charmed decays. The most successful example of this was the decay $D \to \bar{K}\pi$ [3, 4]. This led to the belief that $N_c \to \infty$ was a good approximation in charmed decays. This idea, when carried over to hadronic B decays failed as theory wanted a_2 to be negative [2] while experiments [5] left no doubt that in B decays a_2 was positive.

Recently, it was shown [6] that in the factorization hypothesis all commonly used models of hadronic form factors had difficulty in explaining the polarization data [5, 7, 8] in $B \to \psi K^*$ decay. It was subsequently shown in [9] that even a modest amount ($\sim 10\%$) of nonfactorized contribution made all form factor models consistent with the polarization data. The consequences of nonfactorization in charmed meson decays have recently been explored in [10, 11, 12, 13] and in B decays in [14].

If we use $N_c = 3$, we get from (2) and (3) at the charm scale,

$$a_1 = 1.09 \pm 0.04 \ , \qquad\qquad a_2 = -0.09 \pm 0.05 \ , \tag{6}$$

and at the bottom scale,

$$a_1 = 1.03 \pm 0.01 \ , \qquad\qquad a_2 = 0.10 \pm 0.03 \ , \tag{7}$$

In [12] and [13] it was shown that the inclusion of nonfactorized contributions allows us to define effective a_1 and a_2 and that even a modest nonfactorized contribution in color-suppressed charm decays could lead to $a_2^{eff} \approx -0.5$, a circumstance mitigated by the large value of the ratio $\frac{C_1}{a_2}$. It has not yet been explicitly shown as to what effects are included in a_1^{eff} and a_2^{eff}: Are they complex, and if so, what makes them so? Where do annihilation processes fit in? What role do final state interaction (fsi) play?

2 An Illustrative Example.

We answer all the questions posed at the end of the preceding section with a specific example from charm decays: $D_s^+ \to \phi\pi^+$. The reason for choosing this Cabibbo-favored decay is that it has only one isospin which makes the fsi calculation somewhat simpler. Before embarking on the details, let us introduce the following color Fierz identities,

$$
\begin{aligned}
(\bar{s}c)(\bar{u}d) &= \frac{1}{N_c}(\bar{s}d)(\bar{u}c) + \frac{1}{2}\sum_{a=1}^{8}(\bar{s}\lambda^a d)(\bar{u}\lambda^a c) \\
(\bar{s}d)(\bar{u}c) &= \frac{1}{N_c}(\bar{s}c)(\bar{u}d) + \frac{1}{2}\sum_{a=1}^{8}(\bar{s}\lambda^a c)(\bar{u}\lambda^a d)
\end{aligned}
\tag{8}
$$

where λ^a are the Gell-Mann matrices. We adopt the following short-hand notations for the color-octet current products on the right hand side of (8),

$$
H_w^{(8)} = \frac{1}{2}\sum(\bar{s}\lambda^a c)(\bar{u}\lambda^a d) \ , \qquad \tilde{H}_w^{(8)} = \frac{1}{2}\sum(\bar{s}\lambda^a d)(\bar{u}\lambda^a c) \ . \tag{9}
$$

Using (1) and (8), the decay amplitude (before fsi effects are brought into play) for $D_s^+ \to \phi\pi^+$ is given by,

$$
A(D_s^+ \to \phi\pi^+) = \tilde{G}_F\left\{a_1\left\langle \phi\pi^+|(\bar{s}c)(\bar{u}d)|D_s^+\right\rangle + C_2\left\langle \phi\pi^+|H_w^{(8)}|D_s^+\right\rangle\right\} \ . \tag{10}
$$

While the matrix element of $H_w^{(8)}$ is completely nonfactorized, the first term in (10) includes a (i) factorized (spectator) term, (ii) any nonfactorized contributions in addition to the factorized (spectator) term and (iii) a W-annihilation term which in turn has a factorized and a nonfactorized part. These individual contributions to the decay amplitude are parametrized as follows:

$$
\begin{aligned}
\left\langle \phi\pi^+|(\bar{s}c)(\bar{u}d)|D_s^+\right\rangle^{fact} &= f_\pi(2m_\phi)\varepsilon.p_{D_s}A_0^{D_s\phi}(m_\pi^2) \ , \\
\left\langle \phi\pi^+|(\bar{s}c)(\bar{u}d)|D_s^+\right\rangle^{nf} &\equiv f_\pi(2m_\phi)\varepsilon.p_{D_s}A_0^{(1)nf} \ , \\
\left\langle \phi\pi^+|(\bar{s}c)(\bar{u}d)|D_s^+\right\rangle^{ann} &\equiv f_{D_s}(2m_\phi)\varepsilon.p_{D_s}A_0^{ann} \ , \\
\left\langle \phi\pi^+|H_w^{(8)}|D_s^+\right\rangle &\equiv f_\pi(2m_\phi)\varepsilon.p_{D_s}A_0^{(8)nf} \ .
\end{aligned}
\tag{11}
$$

Here we have used the form factor notation of [3] while $A_0^{(1)nf}$ and $A_0^{(8)nf}$ were introduced in [12]. The superscript 'ann' stands for annihilation and ε is the polarization four-vector for the ϕ. Putting it all together, one can define an effective a_1 as follows,

$$
A(D_s^+ \to \phi\pi^+) = \tilde{G}_F a_1^{eff} f_\pi(2m_\phi)\varepsilon.p_{D_s}A_0^{D_s\phi}(m_\pi^2) \ , \tag{12}
$$

where

$$a_1^{eff} = a_1 \left\{ 1 + \frac{A_0^{(1)nf}}{A_0^{D_s\phi}(m_\pi^2)} + \frac{C_2}{a_1}\frac{A_0^{(8)nf}}{A_0^{D_s\phi}(m_\pi^2)} + \frac{f_{D_s}}{f_\pi}\frac{A_0^{ann}}{A_0^{D_s\phi}(m_\pi^2)} \right\} . \tag{13}$$

Up to this stage, all quantities are taken to be real, including A_0^{ann}. Complex amplitudes will emerge as the result of fsi at the hadronic level.

Consider now the final state interactions. For illustrative purposes we consider a two-channel model which is adequate to illustrate our ideas. Consider an inelastic coupling of $\phi\pi$ channel with G-parity even, to the G-parity even eigenstate of $\bar{K}^0 K^{*+}$ and $\bar{K}^{*0}K^+$. Channel $\phi\pi$ will couple, among others, to $\eta\rho$ and $\eta'\rho$ channels also. Our intention is not to calculate numerically the effect of these channels but to illustrate how fsi enter the description. Both of these decays,$D_s^+ \to \bar{K}^0 K^{*+}$ and $D_s^+ \to \bar{K}^{*0}K^+$, are color-suppressed. Following an analogous procedure to the one that led us to (12), we find,

$$A(D_s^+ \to \bar{K}^0 K^{*+}) = \tilde{G}_F a_2^{eff} f_K (2m_{K^*})\varepsilon.p_{D_s} A_0^{D_s K^*}(m_K^2) , \tag{14}$$

where

$$a_2^{eff} = a_2 \left\{ 1 + \frac{B_0^{(1)nf}}{A_0^{D_s K^*}(m_K^2)} + \frac{C_1}{a_2}\frac{B_0^{(8)nf}}{A_0^{D_s K^*}(m_K^2)} + \frac{a_1}{a_2}\frac{f_{D_s}}{f_K}\frac{B_0^{ann}}{A_0^{D_s K^*}(m_K^2)} \right\} . \tag{15}$$

Here $B_0^{(1)nf}$, $B_0^{(8)nf}$ and B_0^{ann} are the analogues of $A_0^{(1)nf}$, $A_0^{(8)nf}$ and A_0^{ann} of (11). Similarly,

$$A(D_s^+ \to \bar{K}^{*0} K^{+}) = \tilde{G}_F \hat{a}_2^{eff} f_{K^*} (2m_{K^*})\varepsilon.p_{D_s} F_1^{D_s K}(m_{K^*}^2) \tag{16}$$

where

$$\hat{a}_2^{eff} = a_2 \left\{ 1 + \frac{\hat{B}_0^{(1)nf}}{F_1^{D_s K}(m_{K^*}^2)} + \frac{C_1}{a_2}\frac{\hat{B}_0^{(8)nf}}{F_1^{D_s K}(m_{K^*}^2)} + \frac{a_1}{a_2}\frac{f_{D_s}}{f_{K^*}}\frac{\hat{B}_0^{ann}}{F_1^{D_s K}(m_{K^*}^2)} \right\} . \tag{17}$$

The hatted quantities here refer to the decay channel $\bar{K}^{*0}K^+$.

Now the eigenstates of G-parity are [15],

$$|K^*K\rangle_{S,A} = \frac{1}{\sqrt{2}}\left\{ |K^{*+}\bar{K}^0\rangle \pm |K^+\bar{K}^{*0}\rangle \right\} , \tag{18}$$

where the symmetric (antisymmetric) combination has G-parity even (odd). Thus it is only $|K^*K\rangle_S$ that couples to $\phi\pi$. We note here one further point regarding the annihilation term in (15) and (17). The factorized part of the annihilation term in (15) is $\left\langle \bar{K}^0 K^{*+}|(\bar{u}d)|0\right\rangle$ $\langle 0|(\bar{s}c)|D_s^+\rangle$ which is proportional to the matrix element of the divergence of the axial part of $(\bar{u}d)$ current. Now, if the hadronic weak currents are only of the first class kind then the axial current has G-parity odd. As the symmetric state, $|K^*K\rangle_S$, has G-parity even, it requires that the factorized part of $\hat{B}_0^{ann}$ in (17) be equal in magnitude and opposite in sign to the factorized part of B_0^{ann} in (15). However, nonfactorized annihilation terms (for example, when the intermediate state in the direct channel is not a hadronic vacuum but a multigluonic state [16]) will frustrate this argument.

We, next, set up a coupled channel fsi between the decays $D_s^+ \to \phi\pi^+$ and $D_s^+ \to (K^*K)_S$ following the formalism described in [17]. Though the method of unitarization, the K-matrix method which amounts to retaining only the on-shell contribution from re-scattering loops, is not unique, it serves adequately to describe our ideas.

We simplify our notations further by using the following short-hand notations for the thus far real amplitudes,

$$A(D_s^+ \to \phi\pi^+) \equiv \varepsilon.p_{D_s} A^{\phi\pi} \ , \tag{19}$$

with

$$A^{\phi\pi} = \tilde{G}_F a_1^{eff} f_\pi (2m_\phi) A_0^{D_s\phi}(m_\pi^2) \ , \tag{20}$$

and

$$A(D_s^+ \to (K^*K)_S) \equiv \varepsilon.p_{D_s} A^{K^*K} \ , \tag{21}$$

where

$$A^{K^*K} = \tilde{G}_F \frac{(2m_{K^*})}{\sqrt{2}} \left\{ a_2^{eff} f_K A_0^{D_sK^*}(m_K^2) + \hat{a}_2^{eff} f_{K^*} F_1^{D_sK}(m_{K^*}^2) \right\} \ . \tag{22}$$

The two amplitudes, (19) and (21), couple via fsi and get unitarized. The unitarized decay amplitudes, $\boldsymbol{A}^U$, are given by [17],

$$\boldsymbol{A}^U = \left(\mathbf{1} - i\boldsymbol{k}^3 \boldsymbol{K} \right)^{-1T} \boldsymbol{A} \ , \tag{23}$$

where $\boldsymbol{A}$ is a column with entries $A^{\phi\pi}$ and A^{K^*K}, $\boldsymbol{k}^3$ is a diagonal matrix with entries k_1^3 and k_2^3, k_1 and k_2 being the center of mass momenta in the channels $\phi\pi$ and K^*K respectively and $\boldsymbol{K}$ is the symmetric, real (2×2) K-matrix,

$$\boldsymbol{K} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \ , \tag{24}$$

where a, b and c are assumed to be constants with dimensions GeV^{-3}. Note the appearance of $\boldsymbol{k}^3$ as the appropriate threshold factor for P-waves in (23).

The parameters of the K-matrix could be evaluated from the knowledge of the coupled channel scattering problem. In absence of this information, they remain undetermined in our case. Though, for our purposes the knowledge of the numerical values of the K-matrix is not necessary, we have ventured an estimate of the elements of the K-matrix later.

Carrying out the unitarization of the decay amplitude as indicated in (23), we obtain a unitarized $A^{U,\phi\pi}$ which is complex and given by,

$$A^U(D_s^+ \to \phi\pi^+) = \tilde{G}_F a_1^{U,eff} f_\pi \varepsilon.p_{D_s} (2m_\phi) A_0^{D_s\phi}(m_\pi^2) \ , \tag{25}$$

where

$$a_1^{U,eff} = \frac{a_1^{eff}}{\Delta} \left\{ 1 - ik_2^3 c + i \frac{m_{K^*}}{\sqrt{2} m_\phi} k_2^3 b \left(\frac{a_2^{eff}}{a_1^{eff}} \frac{f_K}{f_\pi} \frac{A_0^{D_sK^*}(m_K^2)}{A_0^{D_s\phi}(m_\pi^2)} + \frac{\hat{a}_2^{eff}}{a_1^{eff}} \frac{f_{K^*}}{f_\pi} \frac{F_1^{D_sK}(m_{K^*}^2)}{A_0^{D_s\phi}(m_\pi^2)} \right) \right\} \ , \tag{26}$$

with $\Delta = \det(\mathbf{1} - i\boldsymbol{k}^3\boldsymbol{K})$.

If the fsi were elastic, $b = c = 0$ and $\Delta = 1 - ik_1^3 a$, we would have obtained

$$a_1^{U,eff} = \frac{a_1^{eff}}{\sqrt{1 + k_1^6 a^2}} e^{i\delta} \ , \tag{27}$$

where $\delta = \tan^{-1}(ak_1^3)$ is the elastic P-wave $\phi\pi$ scattering phase.

Though numerical calculations is not the intent of this paper, a rough estimate of some of the K-matrix elements can be obtained in the manner done in [17]. For example, the off-diagonal T-matrix element,T_{12}, representing the inelastic process $\phi\pi \to (\bar{K}^0 K^{*+} + \bar{K}^{*0} K^+)$, in the K-exchange approximation, is given by,

$$T_{12}(W, \theta) = 2g_{VPP}^2 \frac{\epsilon_\phi \cdot p_k \epsilon_{K^*} \cdot p_\pi}{(p_{K^*} - p_\pi)^2 - m_K^2} \tag{28}$$

where $W = m_{D_s}$ is the center of mass energy and we have used an SU(3)-symmetric Vector-Pseudoscalar-Pseudoscalar (VPP) coupling g_{VPP} given by [18]

$$\frac{g_{VPP}^2}{4\pi} \simeq 3.3 \ . \tag{29}$$

The fact that the decaying particle has spin zero imposes simplifying helicity constraints on the vector particles in the process $\phi\pi \to (K^* K)_S$. As the orbital angular momentum, $\vec{L}$, is orthogonal to the scattering plane, it cannot have a component in the plane of scattering. This forces the helicities of ϕ and K^* to be zero in the re-scattering process. We can then project out $J = 0$ amplitude with $\lambda_\phi = 0, \lambda_{K^*} = 0$ from (28) by using

$$(T_{12}(W, \theta))_{\lambda_\phi, \lambda_{K^*}} = \sum_J (2J+1) d^J_{\lambda_\phi, \lambda_{K^*}}(\theta) T^J_{\lambda_\phi, \lambda_{K^*}}(W) \tag{30}$$

Projecting $(T_{12}^{J=0})_{00}$ from (28), we obtain

$$\begin{aligned}(T_{12}^{J=0})_{00} = \frac{2g_{VPP}^2}{m_\phi m_{K*}} \{ &(E_K E_\pi + \frac{1}{3} E_{K*} E_\phi) Q_0(z) \\ &- \frac{1}{pp'} (p'^2 E_\pi E_\phi + p^2 E_K E_{K*}) Q_1(z) \\ &+ \frac{2}{3} E_\phi E_{K*} Q_2(z) \}\end{aligned} \tag{31}$$

where p and p' are the magnitudes of 3-momenta in the center of mass of $\phi\pi$ and $K^* K$ systems at $W = m_{D_s}$ and $Q_i(z)$are the Legendre functions of the Second Kind with the argument z given by

$$z = \frac{1}{2pp'} \{ 2E_{K*} E_\pi + m_K^2 - m_{K*}^2 - m_\pi^2 \} \tag{32}$$

Finally , we relate $(T_{12}^{J=0})_{00}$ to the off-diagonal K-matrix, K_{12}, through

$$(T_{12}^{J=0})_{00} = 8\pi W k_1 K_{12} k_2 \tag{33}$$

where k_1 and k_2 are the eigen-momenta in the two channels and appear due to the P-wave nature of scattering in $J = 0$ state. Numerically we obtain

$$(T_{12}^{J=0})_{00} = 1.58 g_{VPP}^2, \qquad K_{12} \equiv b = 2.73 GeV^{-3} \tag{34}$$

By a similar technique, one can calculate K_{22} through π exchange in the diagonal channel $(K^*K)_S \to (K^*K)_S$ which goes via $K^{*+}\bar{K}^0 \to \bar{K}^{*0}K^+$. We obtain

$$(T_{22}^{J=0})_{00} = 0.99 g_{VPP}^2, \qquad K_{22} \equiv c = 1.78 GeV^{-3} \tag{35}$$

These parameters, b and c, are quite sizable. Their effect in the unitarization appears in dimensionless products of form $k_1^3 b, k_2^3 b$ and $k_2^3 c$ which are numerically 0.98, 0.87 and 0.57 respectively. It is harder to estimate $K_{11}(= a)$ as the elastic $\phi\pi$ scattering is an OZI-violating process. Thus, though our model is crude, it appears very likely that inelastic fsi could play an important role in $D_s^+ \to \phi\pi$ decay.

A similar expression to (26) can be written down for $D_s^+ \to K^{*+}\bar{K}^0$ (and for $D_s^+ \to K^+\bar{K}^{*0}$) which would define $a_2^{U,eff}$ and $\hat{a}_2^{U,eff}$.

One should view (25) as the defining equation for $a_1^{U,eff}$ which includes all conceivable physics, is process-dependent and complex.If we view $a_1^{U,eff}$ and $a_2^{U,eff}$ in the manner we are proposing, then the comparison of two body hadronic decays of D and B mesons with semileptonic decays which in past has been claimed [2, 5, 19, 20] to be tests of factorization, becomes merely determinations of $|a_1^{U,eff}|$ and $|a_2^{U,eff}|$.

3 Estimates of $|a_1^{U,eff}|$ and $|a_2^{U,eff}|$ from charm and beauty decay data .

The procedure we have outlined above can be used in defining $a_1^{U,eff}$ and $a_2^{U,eff}$ in, say, $D^0 \to \bar{K}\pi$ decays. There is an added complication here, that of two isospins in the final state. The fsi unitarization has to be carried out in each of the two isospin states separately. Nevertheless, one can define, following the same procedure as we have used for the simpler case of $D_s^+ \to \phi\pi^+$,

$$\begin{aligned} A(D^0 \to K^-\pi^+) &= \tilde{G}_F |a_1^{U,eff}| f_\pi (m_D^2 - m_K^2) F_0^{DK}(m_\pi^2) e^{i\phi_{+-}} \\ A(D^0 \to \bar{K}^0\pi^0) &= \frac{\tilde{G}_F}{\sqrt{2}} |a_2^{U,eff}| f_K (m_D^2 - m_\pi^2) F_0^{D\pi}(m_K^2) e^{i\phi_{00}} \\ A(D^+ \to \bar{K}^0\pi^+) &= A(D^0 \to K^-\pi^+) + \sqrt{2} A(D^0 \to \bar{K}^0\pi^0) \ . \end{aligned} \tag{36}$$

In principle, $|a_1^{U,eff}|$ can be determined by relating $\Gamma(D^0 \to K^-\pi^+)$ to $\Gamma(D^0 \to K^- l^+ \nu)$ and $|a_2^{U,eff}|$ by relating $\Gamma(D^0 \to \bar{K}^0\pi^0)$ to $\Gamma(D^0 \to \pi^- l^+ \nu)$. Finally, $\phi_{+-} - \phi_{00}$ is, in principle, obtainable from $\Gamma(D^+ \to \bar{K}^0\pi^+)$.We determined the products $|a_1^{U,eff}| F_0^D K(m_\pi^2)$ and $|a_2^{U,eff}| F_0^{D\pi}(m_K^2)$ and the relative phase $(\phi_{+-} - \phi_{00})$ from the branching ratios $B(D^0 \to K^-\pi^+)$, $B (D^0 \to \bar{K}^0 \pi^0)$ and $B(D^+ \to \bar{K}^0\pi^+)$ [21] with the result:

$$\begin{aligned} D \to \bar{K}\pi : \qquad & |a_1^{U,eff}| F_0^{DK}(m_\pi^2) = \ 0.767 \pm 0.014 \\ & |a_2^{U,eff}| F_0^{D\pi}(m_K^2) = \ 0.593 \pm 0.038 \\ & cos(\phi_{+-} - \phi_{00}) = \ -0.867 \pm 0.089. \end{aligned} \tag{37}$$

If we use the experimental determinations [21] of $F_0^{DK}(0)$ and $F_0^{D\pi}(0)$ from semileptonic decays (assuming monopole extrapolation),

$$\begin{aligned} F_0^{DK}(0) &= \quad 0.75 \pm 0.02 \pm 0.02 \qquad [21] \\ F_0^{D\pi}(0)/F_0^{DK}(0) &= \quad 1.0^{+0.3}_{-0.2} \pm 0.04 \qquad [22] \\ &= \quad 1.3 \pm 0.2 \pm 0.1 \qquad [23] \end{aligned} \tag{38}$$

we obtain,

$$\begin{aligned} |a_1^{U,eff}| &= \quad 1.02 \pm 0.04 \\ |a_2^{U,eff}| &= \quad (0.76^{+0.26}_{-0.16}), \qquad (0.58 \pm 0.08) \end{aligned} \tag{39}$$

In $a_2^{U,eff}$ above, the two values correspond to the two values of the form factor ratio $F_0^{D\pi}(0)/F_0^{DK}(0)$ given in (38) respectively. We have used a monopole extrapolation with pole mass 2.47 GeV [3] in calculating $F_0^{D\pi}(m_K^2)$.

A similar analysis of the branching ratios in $D \to \bar{K}\rho$ and $\bar{K}^*\pi$leads to:

$$D \to \bar{K}\rho : \qquad \begin{aligned} |a_1^{U,eff}|F_1^{DK}(m_\rho^2) &= \quad 1.097 \pm 0.069 \\ |a_2^{U,eff}|A_0^{D\rho}(m_K^2) &= \quad 0.672 \pm 0.055 \\ cos(\phi_{+-} - \phi_{00}) &= \quad -1.046 \pm 0.205 \end{aligned} \tag{40}$$

and

$$D \to \bar{K}^*\pi : \qquad \begin{aligned} |a_1^{U,eff}|A_0^{DK^*}(m_\pi^2) &= \quad 1.138 \pm 0.070 \\ |a_2^{U,eff}|F_1^{D\pi}(m_{K^*}^2) &= \quad 0.747 \pm 0.061 \\ cos(\phi_{+-} - \phi_{00}) &= \quad -0.926 \pm 0.166 \end{aligned} \tag{41}$$

From the form factors at $q^2 = 0$ listed in [21] we can calculate all the form factors needed by using monopole extrapolation for all of them except $A_0^{D\rho}(m_K^2)$ for which we adopt the theoretical value given in [3] . The resulting $a_1^{U,eff}$ and $a_2^{U,eff}$ are :

$$D \to \bar{K}\rho : \qquad \begin{aligned} |a_1^{U,eff}| &= \quad 1.27 \pm 0.09 \\ |a_2^{U,eff}| &= \quad 0.93 \pm 0.08 \end{aligned} \tag{42}$$

and

$$D \to \bar{K}^*\pi : \qquad \begin{aligned} |a_1^{U,eff}| &= \quad 1.76 \pm 0.23 \\ |a_2^{U,eff}| &= \quad (0.8^{+0.27}_{-0.17}), \quad (0.61 \pm 0.09) \end{aligned} \tag{43}$$

The two values of $a_2^{U,eff}$ correspond to the two values of the ratio $F_0^{D\pi}(0)/F_0^{DK}(0)$ respectively, given in (38).

We end with a determination of the process-dependent $|a_1^{U,eff}|$ in $D_s^+ \to \eta\pi^+$, $\eta'\pi^+$, $\eta\rho^+$ and $\eta'\rho^+$ and $B^0 \to D^-\pi^+$ and $D^-\rho^+$ from hadronic and semileptonic data. For D_s^+ decays, we provide a calculation for $D_s^+ \to \eta\pi^+$ and $\eta\rho^+$ to illustrate the method, details of which may be found in Refs. [20] and [24].

The defining equation for $a_1^{U,eff}$ in $D_s^+ \to \eta\pi^+$ and $\eta\rho^+$ decay amplitudes is obtained by simply replacing a_1 in the expression for the factorized amplitude by $a_1^{U,eff}$ as in (25). Thus

$$A(D_s^+ \to \eta\pi^+) = \tilde{G}_F C_\eta (a_1^{U,eff})_{\eta\pi^+} f_\pi (m_{D_s}^2 - m_\eta^2) F_0^{D_s\eta}(m_\pi^2) \ , \tag{44}$$

where $F_0^{D_s\eta}$ is the relevant form factor [3], and in terms of flavor singlet-octet mixing angle θ_P,

$$C_\eta = \sqrt{\frac{2}{3}} \left(\cos\theta_P + \frac{1}{\sqrt{2}} \sin\theta_P \right) \ . \tag{45}$$

The resulting decay rate is,

$$\Gamma(D_s^+ \to \eta\pi^+) = \frac{\tilde{G}_F^2}{16\pi m_{D_s}^3} |(a_1^{U,eff})_{\eta\pi^+}|^2 \left(C_\eta f_\pi (m_{D_s}^2 - m_\pi^2) \right)^2 \lambda(m_{D_s}^2, m_\eta^2, m_\pi^2) |F_1^{D_s\eta}(0)|^2 \ , \tag{46}$$

where $\lambda(x,y,z) = (x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)^{1/2}$ and we have used $F_0^{D_s\eta}(m_\pi^2) \approx F_0^{D_s\eta}(0) = F_1^{D_s\eta}(0)$. Similarly,

$$A(D_s^+ \to \eta\rho^+) = \tilde{G}_F C_\eta (a_1^{U,eff})_{\eta\rho^+} (2 m_\rho f_\rho) \varepsilon^* . p_{D_s} F_1^{D_s\eta}(m_\rho^2) \ , \tag{47}$$

which leads to

$$\Gamma(D_s^+ \to \eta\rho^+) = \frac{\tilde{G}_F^2}{16\pi m_{D_s}^3} |(a_1^{U,eff})_{\eta\rho^+}|^2 \left(C_\eta f_\rho \right)^2 \lambda^3(m_{D_s}^2, m_\eta^2, m_\rho^2) \frac{|F_1^{D_s\eta}(0)|^2 \Lambda_1^{4n}}{(\Lambda_1^2 - m_\rho^2)^{2n}} \ , \tag{48}$$

where $n = 1(2)$ for a monopole (dipole) extrapolation of the form factor. Λ_1 is taken to be 2.11 GeV, the D_s^* mass.

Consider now the semileptonic decay rate for $D_s^+ \to \eta e^+ \nu$ [20, 24], which can be written as,

$$\Gamma(D_s^+ \to \eta e^+ \nu) = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} C_\eta^2 |F_1^{D_s\eta}(0)|^2 \Lambda_1^{4n} I_n(m_{D_s}, m_\eta, \Lambda_1) \ , \tag{49}$$

where

$$I_n(m_{D_s}, m_\eta, \Lambda_1) = \int_0^{(m_{D_s} - m_\eta)^2} dq^2 \frac{\lambda^3(m_{D_s}^2, m_\eta^2, q^2)}{(q^2 - \Lambda_1^2)^{2n}} \ . \tag{50}$$

¿From (46), (48) and (49), we can construct the ratios the ratios $\frac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \eta e^+\nu)}$ and $\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \eta e^+\nu)}$ involving the unknowns $|(a_1^{U,eff})_{\eta\pi^+}|$ and $|(a_1^{U,eff})_{\eta\rho^+}|$. By equating these theoretical ratios to the experimental ones, we can determine $|a_1^{U,eff}|$ in these two decays. A similar method can be applied to decays involving η' in the final state.

We now turn to the experimental results we have used. Recently, CLEO collaboration has measured [25] the following ratios,

$$\begin{aligned} \frac{\Gamma(D_s^+ \to \eta e^+ \nu)}{\Gamma(D_s^+ \to \phi e^+ \nu)} &= 1.24 \pm 0.12 \pm 0.15 \ , \\ \frac{\Gamma(D_s^+ \to \eta' e^+ \nu)}{\Gamma(D_s^+ \to \phi e^+ \nu)} &= 0.43 \pm 0.11 \pm 0.07 \ . \end{aligned} \tag{51}$$

If we combine this with the following measured ratios,

$$\frac{\Gamma(D_s^+ \to \phi\pi^+)}{\Gamma(D_s^+ \to \phi e^+\nu)} = 0.54 \pm 0.05 \pm 0.04 \qquad [21] \ , \tag{52}$$

$$\frac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \phi\pi^+)} = 0.54 \pm 0.09 \pm 0.06 \qquad [26] \ , \tag{53}$$

$$\frac{\Gamma(D_s^+ \to \eta'\pi^+)}{\Gamma(D_s^+ \to \phi\pi^+)} = 1.2 \pm 0.15 \pm 0.11 \qquad [26] \ , \tag{54}$$

$$\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \phi\pi^+)} = 2.86 \pm 0.38^{+0.36}_{-0.38} \qquad [27] \ , \tag{55}$$

and

$$\frac{\Gamma(D_s^+ \to \eta'\rho^+)}{\Gamma(D_s^+ \to \phi\pi^+)} = 3.44 \pm 0.62^{+0.44}_{-0.46} \qquad [27] \ . \tag{56}$$

We obtain the following experimental ratios:

$$\frac{\Gamma(D_s^+ \to \eta\pi^+)}{\Gamma(D_s^+ \to \eta e^+\nu)} = 0.81 \pm 0.23 \ , \tag{57}$$

$$\frac{\Gamma(D_s^+ \to \eta'\pi^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)} = 5.17 \pm 1.86 \ , \tag{58}$$

$$\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \eta e^+\nu)} = 4.27 \pm 1.13 \ , \tag{59}$$

and

$$\frac{\Gamma(D_s^+ \to \eta'\rho^+)}{\Gamma(D_s^+ \to \eta' e^+\nu)} = 14.81 \pm 5.81 \ , \tag{60}$$

The errors here are probably overestimated as we propagated all errors as if they were independent while some systematic errors in the products of ratios would cancel.

Confronting the theoretical ratios to the experimental ones shown in (57) - (60) we have evaluated the following (we have used $V_{ud} = 0.975$, $f_\pi = 130.7$ MeV and $f_\rho = 216.0$ MeV):

$$\begin{aligned} |(a_1^{U,eff})_{\eta\pi^+}| &= 0.89 \pm 0.13 \qquad (n \ = \ 1) \\ &= 1.08 \pm 0.15 \qquad (n \ = \ 2) \ , \end{aligned} \tag{61}$$

$$\begin{aligned} |(a_1^{U,eff})_{\eta'\pi^+}| &= 1.56 \pm 0.28 \qquad (n \ = \ 1) \\ &= 1.68 \pm 0.30 \qquad (n \ = \ 2) \ , \end{aligned} \tag{62}$$

$$
\begin{aligned}
|(a_1^{U,eff})_{\eta\rho^+}| &= 1.49 \pm 0.20 \qquad (n\ =\ 1) \\
&= 1.55 \pm 0.20 \qquad (n\ =\ 2)\ ,
\end{aligned} \tag{63}
$$

$$
\begin{aligned}
|(a_1^{U,eff})_{\eta'\rho^+}| &= 2.77 \pm 0.55 \qquad (n\ =\ 1) \\
&= 2.60 \pm 0.51 \qquad (n\ =\ 2)\ .
\end{aligned} \tag{64}
$$

In most cases $|a_1^{U,eff}|$ has risen for a dipole form factor compared to the monopole, except for $D_s^+ \to \eta'\rho^+$. The reason is that the hadronic rate, $\Gamma(D_s^+ \to \eta'\rho^+)$, rises more than the semileptonic rate, $\Gamma(D_s^+ \to \eta' e^+\nu)$, when one goes from the monopole to a dipole form factor in contrast to the other cases.

For $B^0 \to D^-\pi^+$ and $D^-\rho^+$, we define $a_1^{U,eff}$ via,

$$
A(B^0 \to D^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} \left(a_1^{U,eff}\right)_{D\pi} f_\pi \left(m_B^2 - m_D^2\right) F_1^{BD}(0)\ , \tag{65}
$$

$$
A(B^0 \to D^-\rho^+) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} \left(a_1^{U,eff}\right)_{D\rho} (2m_\rho f_\rho)\, \varepsilon.p_B F_1^{BD}(m_\rho^2)\ , \tag{66}
$$

where we have used $F_0^{BD}(m_\pi^2) \approx F_0^{BD}(0) = F_1^{BD}(0)$. The hadronic rates are calculated from the above two equations and the semileptonic rate from an analogous formula to (49). For the experimental branching ratios, we used Ref. [21] and evaluated $|a_1^{U,eff}|$ for $B \to D\pi$ and $D\rho$ decays. We used four different extrapolations for the form factor $F_1^{BD}(q^2)$: (i) monopole and (ii) dipole with pole mass 6.34 GeV [3], (iii) an exponential form as in Ref. [28],

$$
F_1^{BD}(t = q^2) \propto \exp\left\{0.025(t - t_m)\right\} \tag{67}
$$

where $t_m = (m_B - m_D)^2$ in GeV^{-2}, and (iv) a form advocated in [2],

$$
F_1^{BD}(t = q^2) \propto \frac{2}{y+1} \exp\left\{-\beta \frac{y-1}{y+1}\right\} \tag{68}
$$

where $\beta = 2\rho^2 - 1$ with $\rho = 1.19$ [2] and,

$$
y \equiv v.v' = \frac{(m_B^2 + m_D^2 - q^2)}{2 m_B m_D} \tag{69}
$$

The resulting $|a_1^{U,eff}|$ for each of the four form factor extrapolations are listed below.
For $B \to D\pi$:

$$
\begin{aligned}
|a_1^{U,eff}| &= 0.91 \pm 0.05 \qquad \text{(monopole)} \\
&= 1.0 \pm 0.06 \qquad \text{(dipole)} \\
&= 0.91 \pm 0.05 \qquad \text{(using (67))} \\
&= 1.0 \pm 0.06 \qquad \text{(using (68))}
\end{aligned} \tag{70}
$$

For $B \to D\rho$:

$$
\begin{aligned}
|a_1^{U,eff}| &= 0.91 \pm 0.08 \qquad \text{(monopole)} \\
&= 0.99 \pm 0.09 \qquad \text{(dipole)} \\
&= 0.91 \pm 0.08 \qquad \text{(using (67))} \\
&= 0.99 \pm 0.09 \qquad \text{(using (68))}
\end{aligned} \tag{71}
$$

4 Summary

In summary, the effective, and unitarized a_1 and a_2 which are defined by the following prescription: The true decay amplitude is given by replacing a_1 and a_2 by $a_1^{U,eff}$ and $a_2^{U,eff}$ respectively in the factorized (spectator) amplitude. Defined in this manner, as we have shown systematically how these effective parameters get contributions from annihilation and nonfactorizable processes as well as the final state interactions. As these effective parameters are process-dependent, the purported test of factorization that compares the hadronic rate to the semileptonic should be used, instead, as a tool to determine the modulus of these effective parameters.

We determined $|a_1^{U,eff}|$ and $|a_2^{U,eff}|$ in $D \to \bar{K}\pi, \bar{K}\rho$and $\bar{K}^*\pi$ decays using experimental input on formfactors (with monopole extrapolation) as much as possible. The values of these parameters, particularly $|a_2^{U,eff}|$, imply large departures from the factorization expectation when compared with a_1 and a_2 given by (6) with $N_c = 3$.

For $D_s^+ \to \eta'\pi^+$, $\eta\rho^+$ and $\eta'\rho^+$, $a_1^{U,eff}$ was found to be significantly different from a_1 of eq.(6) signifying that the simple factorization prescription would not apply to these cases. For $B^0 \to D^-\pi^+$ and $D^-\rho^+$, however, we found $|a_1^{U,eff}|$ to be not much different from a_1 of eq.(7), especially for the form factor extrapolations given by a dipole and eq.(68), signifying that effects such as annihilation, nonfactorization and fsi play a less significant role in hadronic B decays.

Finally, as emphasized in Refs. [9] and [11], effective a_1 and a_2 can be defined only for those decays whose amplitudes involve a single Lorentz scalar structure. Thus they can not be defined for decays of D and B mesons involving two vector particles in the final state. Consequently, our analysis applies only to those cases where the decay amplitudes involve a single Lorentz scalar structure.

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References

[1] R. Rückl, Habilitationsschrift, Univ. of Munich (1983).

[2] M. Neubert, V. Rieckert, B.Stech and Q. P. Xu, in: Heavy Flavours (World Scientific, Singapore, 1992) eds. A. J. Buras and M. Lindner.

[3] M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C34** 103 (1987); M. Wirbel, B. Stech and M. Bauer, Z. Phys. **C29**, 637 (1985).

[4] A. N. Kamal and T. N. Pham, Phys. Rev. **D50**, 6849 (1994).

[5] CLEO collaboration, M. S. Alam et al. , Phys. Rev. **D50**, 43 (1994).

[6] M. Gourdin, A. N. Kamal and X. Y. Pham, Phys. Rev. Lett. **73**, 3355 (1994).

[7] CDF Collaboration, F.Abe etal., Phys.Rev.Lett.**75**,3608 (1995).

[8] ARGUS Collaboration, H. Albrecht et al., Phys. Lett. **B340**, 217 (1994).

[9] A. N. Kamal and A. B. Santra, University of Alberta Report No. Alberta Thy-31-94 (1994).To be published in Z. Phys. **C** .

[10] H. -Y Cheng, Phys. Lett. **B335**, 428 (1994).

[11] H. -Y Cheng, Taipei Report No. IP-ASTP-04-95 (1995).

[12] A. N. Kamal and A. B. Santra, University of Alberta Report No. Alberta Thy-1-95 (1995).To be published in Z. Phys. **C** .

[13] A. N. Kamal and A. B. Santra, T.Uppal and R. C. Verma, University of Alberta Report No. Alberta Thy-08-95 (1995).To be published in Phys.Rev.**D** .

[14] J. M. Soares, Phys. Rev. **D51**, 3518 (1995).

[15] A. N. Kamal, N. Sinha and R. Sinha, Z. Phys. **C41**, 207 (1988).

[16] H. J. Lipkin, in Heavy Quark Physics, AIP conf proceedings 196, eds: Persis S. Drell and David L. Rubin, AIP, New York (1989).

[17] A. N. Kamal, Int. J. Mod. Phys. **A7**, 3515 (1992).

[18] R. L. Thews and A. N. Kamal, Phys. Rev. **D32**, 810 (1985).

[19] D. Bortoletto and S. Stone, Phys. Rev. Lett. **65**, 2951 (1990)

[20] A. N. Kamal, Q. P. Xu and A. Czarnecki, Phys. Rev. **D48**, 5215 (1993).

[21] Particle Data Group, L. Montanet et al., Phys. Rev. **D50**, 1173 (1994).

[22] MARK III Collaboration, J. Adler etal., Phys.Rev.Lett. **62**, 1821 (1989).

[23] CLEO Collaboration, M.S. Alam etal., Phys.Rev.Lett. **71**, 1311 (1993)

[24] R. C. Verma, A. N. Kamal and M. P. Khanna, Z. Phys. **C65**, 255 (1995).

[25] CLEO Collaboration, G. Brandenberg et al., phys. Rev. Lett. **75**, 3804 (1995).

[26] CLEO Collaboration, J. Alexander et al., Phys. Rev. Lett. **68**, 1275 (1992).

[27] CLEO Collaboration, P. Avery et al., Phys. Rev. Lett. **68**, 1279 (1992).

[28] N. Isgur, D. Scora, B. Grinstein and M. B. Wise, Phys. Rev. **D39**, 799 (1989).